Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 2, pp. 437–440, 1996

0017–9310/96 $9.50+0.00

0017-9310(95)00119-0

TECHNICAL NOTE

Upstream migration of heat during combined convection in a horizontal parallel plate duct

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(*Received* 10 *June* 1994 *and in final form* 23 *February* 1995)

INTRODUCTION

An extensive review of recent analytical and experimental developments in combined convection in internal flows has been published by Aung [1]. It has been observed, both experimentally and theoretically (see, for example, refs. [2–5]), that steady laminar recirculations may occur in a vertical parallel plate duct for certain combinations of the relevant parameters. In these flows, the fluid was assumed to be flowing along the duct with a fully developed velocity profile, i.e. the Poiseuille flow, at a constant temperature, which is the same as that of the walls of the duct. Then, at a specified location along the duct, the temperature of the wall was suddenly increased or decreased to another constant value. These recirculations may occur either adjacent to the walls of the duct or at the duct centre depending upon whether the walls of the duct are heated or cooled. Ingham *et al.* [2–4] performed numerical calculations which entirely predicted these flow recirculations, and the resulting fully developed flows at a large distance along the duct matched the analytical solution of Aung and Worku [6]. However, there was an upstream migration of heat from the location where the temperature change occurred. The length of this upstream migration of heat diminished as the Reynolds number became larger. In the limit as the Reynolds number becomes infinite, there is no penetration of heat upstream of the location where there is a temperature change on the surface of the duct. This phenomenon was thoroughly investigated by Ingham *et al.* [2, 3].

When the duct is horizontal, the buoyancy force acts transversely to the main fluid flow direction in the duct. Ingham *et al.* [7] have extensively investigated this situation numerically over a large range of values of the governing parameters. In particular, they investigated the situation when at some location along the lower wall of the duct the temperature of the surface is suddenly increased. They found as the Reynolds number becomes larger, with the Prandtl number and the ratio of the Grashof to the square of the Reynolds number held fixed, that the recirculating flow which is present in the vicinity of the temperature change increases in length upstream of the temperature discontinuity. In fact, this length was found to enlarge linearly with increasing values of the Reynolds number. Because of the difficulties encountered by Ingham *et al.* [7] in obtaining numerical results at large values of the Reynolds number, a scale analysis was performed in order to obtain the general upstream flow structure at large values of the Reynolds number, and the results presented in this paper. Essentially, the technique applied is very similar to that developed by Smith [8–10] when investigating constricted channel flows, where the length and pressure scales of the upstream influence were sought as a function of the Reynolds number. The results of this large Reynolds number analysis for this heat transfer problem demonstrate that the solution differs considerably from the classical boundary-layer structure which develops downstream of a leading edge and also from the situation when the duct is vertical.

GOVERNING EQUATIONS

The physical situation under consideration is exactly the same as that investigated numerically by Ingham *et al.* [7], namely steady laminar combined convection of a viscous fluid with velocity components (u, v) confined between two very wide horizontal parallel plates which form the infinite domain $-\infty < x < \infty$, $-a \leqslant y \leqslant a$ (see Fig. 1). The walls of the duct are maintained at the following temperatures:

$$T = T_c \quad -\infty < x < 0 \quad y = -a$$
$$T = T_h \quad 0 \leqslant x < \infty \quad y = -a$$
$$T = T_c \quad -\infty < x < \infty \quad y = +a. \qquad (1)$$

In the upstream region $(x < 0)$, the fluid is fully developed and is at a constant temperature T_c. At the duct exit $(x \to \infty)$, fully developed Poiseuille flow is attained and the temperature profile is a linear function of y, i.e.

$$T = T_h + \frac{(y+a)}{2a}(T_c - T_h). \qquad (2)$$

It is assumed that gravity acts vertically downwards in the negative y direction perpendicular to the surface of the duct, the Boussinesq approximation is invoked and viscous dissipation is neglected.

The fluid and thermal fields within the horizontal two-

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NOMENCLATURE

a	half width of duct	U, V	non-dimensional velocity components in the X and Y directions, equal to $up_e\nu/(2aC)$ and $(Vp_e\nu)/(2aC)$, respectively.
C	pressure gradient		
c_p	specific heat at constant pressure		
g	acceleration due to gravity		
Gr	Grashof number, $g\beta\,(T_h - T_e)\,(2a)^3/\nu^2$		
k	thermal conductivity	Greek symbols	
L	upstream separation length	β	coefficient of thermal expansion
P	non-dimensional pressure, $(p+g\rho_e y)2aC$	δ	thickness of viscous wall layer
p	pressure	Δ	streamwise length scale
Pr	Prandtl number, $\rho\,\nu\,c_p/k$	θ	non-dimensional temperature, $(T-T_e)/(T_h-T_e)$
Re	Reynolds number, $(2a)C/(\rho\nu^2)$		
T	fluid temperature	λ	magnitude of pressure force
x, y	horizontal and vertical coordinates, respectively	ν	kinematic viscosity
		ρ	fluid density.
X, Y	non-dimensional horizontal and vertical coordinates, equal to $x/(2a)$ and $y/(2a)$, respectively		
		Subscripts	
u, v	fluid velocity components in the x and y directions, respectively	e	entry
		h	hot.

dimensional parallel walled duct are governed by the equations of continuity, streamwise and transverse momentum, and energy, which on using the following non-dimensionalization parameters, are given by equations (4)–(7)

$$X = x/2a \quad Y = y/2a \quad U = \frac{up_e\nu}{2aC} \quad V = \frac{vp_e\nu}{2aC} \tag{3}$$

$$P = (p+g\rho_e y)2aC \quad \theta = (T-T_e)/(T_h-T_e)$$

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{4}$$

$$Re\left\{U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right\} = -\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2} \tag{5}$$

$$Re\left\{U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right\} = -\frac{\partial P}{\partial Y}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+\frac{Gr}{Re}\theta \tag{6}$$

$$Re\left\{U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right\} = Pr^{-1}\left\{\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right\}. \tag{7}$$

THE UPSTREAM STRUCTURE FOR $Re \gg 1$

When heat is applied at the lower wall there is an upstream migration of heat caused by the buoyancy force and this acts as a thermal obstruction, which reduces the effective duct width. Figure 2 shows a typical streamline pattern at large values of Re and Gr/Re^2; see ref. [7] Thus, the fluid flow must respond in anticipation of this change of thermal boundary condition and the following analysis is designed to estimate the upstream length and pressure scales associated with the buoyancy-induced migration. In the situation of a constricted channel with isothermal flows, as studied by Smith [8–10], the type of constriction was defined according to its slope. For small constrictions, or dilations, where the slope $\ll \mathbf{O}((Re)^{-3/7})$, there exists virtually no non-linear upstream influence so that viscous wall layer motions develop from the leading edge of the constriction and not upstream [8, 9]. Hence, non-linear effects such as viscous separation upstream of the obstacle cannot occur; however, the present work has much in common with the work of Smith [10], where fluid driven through a channel responds upstream of a severe asymmetric distortion resulting in fluid separation.

In ref. [10] the characteristic Reynolds number is assumed to be large and the channel inlet flow profile is fully developed. The upstream disturbance is caused by the transverse pressure gradient induced by a small core flow displacement. A positive pressure perturbation in the vicinity of one wall results in the thickening of the viscous layer and on the other wall the layer thins in order to conserve mass flux. The large transverse pressure gradient induced is subsequently found to maintain the growth of the core dis-

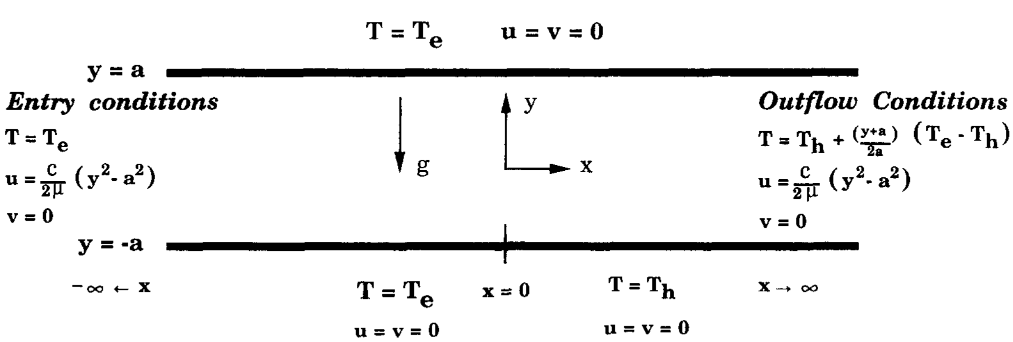

Fig. 1. Schematic diagram of the physical situation.

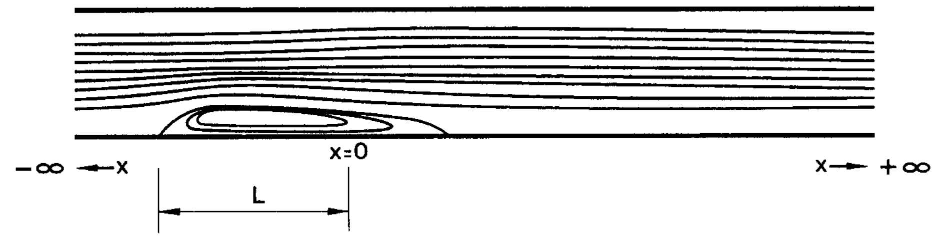

Fig. 2. Typical streamline pattern at large values of Re and Gr/Re^2.

placement, which ultimately induces separation at a distance $\mathbf{O}(a\,(Re)^{1/7})$ upstream of the flow constriction.

In order to determine the upstream length and pressure scales for this heat transfer problem an analysis similar to that developed by Smith [10] is employed. Far upstream the motion is fully developed so that:

$$U = (1-Y^2)/2 \quad V = 0 \quad P = P_e - X \quad \theta = 0 \qquad (8)$$

where P_e is some $\mathbf{O}(1)$ constant. The parameters Gr/Re and Pr are assumed to be constant and the relevant upstream scalings are now sought as functions of the Reynolds number, which is assumed to be very large. These are the thickness of the viscous wall layer, δ, the magnitude of the pressure force, λ, and a streamwise length scale, Δ. Hence, following Smith [10], we assume that the scalings take the form:

$$\lambda \sim (Re)^q \quad \delta \sim (Re)^{-r} \quad \Delta \sim (Re)^s \qquad (9)$$

where q, r and s are constants to be determined. Since the upstream influence exists for $Re \to \infty$, $s \geqslant 0$, also the separation point must be preceded by a change of sign in the streamwise pressure gradient at the lower wall so that $q \geqslant s$, and any viscous layer cannot be thicker than the duct width which implies that $r \geqslant 0$. Introducing the scalings (9) into equations (4)–(7) and seeking a balance between the terms, leads to $q = 1$, $r = 0$ and $s = 1$, so that the upstream scalings become:

$$\lambda \sim Re \quad \delta \sim \mathbf{O}(1) \quad \Delta \sim Re. \qquad (10)$$

Hence, for $Re \gg 1$, the separation point is located at a distance $\mathbf{O}(aRe)$ upstream of the onset of heating.

The scalings developed above differ considerably from the results obtained by Smith [10], where $\Delta \sim (Re)^{-1/7}$, $\delta \sim (Re)^{-2/7}$ and $\lambda \sim (Re)^{3/7}$, and therefore very important differences exist between the present work and that of Smith [10]. In these thermal problems, the upstream influence is implicit in the governing equations, which contrasts to the work of Smith [10], where the flow responds to a geometric change enforced through the boundary conditions. An additional difference is that a physical obstacle induces viscous separation upstream, while a thermally generated reverse flow region is itself a separated region and does not induce any additional upstream separated regions.

The Poiseuille flow profile must develop a distinct nonlinear character prior to the separation point in order for the laminar flow to bypass the recirculation region. In this respect some similarities exist with the constricted channel problem of Smith [10]. Far upstream of the separation point transverse momentum is generated via a small displacement of the core flow, which induces in the transverse direction a pressure gradient and so momentum. Immediately upstream of the separation point both the reversal of the streamwise pressure gradient and viscous wall layers occur.

Separation of the lower wall layer results in a detached shear layer forming around the recirculation region where $U \sim \mathbf{O}(1)$ as opposed to $U \sim \mathbf{O}(\delta)$ in the upstream wall layer.

Correlation of the results of Ingham *et al.* [7] gives the upstream separation length, L, to be

$$L = 2a(8.84 \times 10^{-6}(Gr/Re^2)^{2.30}Re + 0.113\ln(Gr/Re^2) - 0.303) \qquad (11)$$

for $5 \leqslant Re \leqslant 20$, $20 \leqslant Gr/Re^2 \leqslant 50$, but with the restriction that $Gr/Re^2 \geqslant 17$, so that recirculations do exist. Although the numerical results were obtained for moderate values of the Reynolds number, as opposed to large values, there is sufficient agreement between the theory and numerical results to conclude that the upstream reattachment length varies linearly with the Reynolds number. Unfortunately the theory does not predict the height, length or strength of the recirculation zone and hence no further comparisons with the numerical predictions are possible.

CONCLUSIONS

The length and pressure scales associated with the upstream influence of buoyancy have been developed analytically and confirmed by the numerical calculations of Ingham *et al.* [7] The upstream recirculation region predicted in this two-dimensional study results from the inclusion of buoyancy within the governing equations, whereas in the constricted flow problem as considered by Smith [10] the upstream viscous separation is caused by geometrical changes involving only the shape of the duct boundary. Hence, the different scales as obtained by Smith [10] and the present predictions are the result of different mechanisms which generate upstream influences in these flows. In both situations, divergence from Poiseuille flow occurs far upstream of the separation point and follows the form specified by Smith [10], in that core flow displacements induce transverse pressure gradients and momentum. Thus, unequal viscous wall layers develop where the upper and lower layers ultimately separate. A physical change in the duct shape can induce a recirculation but changes in the thermal boundary conditions can also induce a recirculation.

It is concluded, because of the upstream influence caused by the change in thermal boundary conditions, that classical boundary-layer calculations which start at the point of heating are not valid for non-vertical ducts. Hence it is necessary to solve the full governing equations of heat and fluid motion in non-vertical ducts both prior to and after the location where heating takes place even at large values of the Reynolds number.

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